Study of $SU(N)$ spin chains via anomaly and global inconsistency matching

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$SU(2)$ spin chains and Haldane conjecture

Consider the 1D quantum antiferromagnetic spin chain ($J > 0$):

$$H = J \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}.$$

Haldane ('83) showed that the low-energy EFT is the $\mathbb{CP}^1$ NLσM,

$$S = \frac{1}{g^2} \int |(\mathrm{d} + \mathrm{i}a) z|^2 + \frac{\mathrm{i}\theta}{2\pi} \int \mathrm{d}a,$$

with $\theta = 2\pi |\boldsymbol{S}|$ (as $|\boldsymbol{S}| \gg 1$).

- $S = 1, 2, 3, \dots \Rightarrow$ The system has the mass gap (Polyakov, '75).
- $S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots \Rightarrow$ The system is gapless, especially described by $SU(2)_1$ WZW model (Affleck, Haldane, '87).

Generalization of the Haldane conjecture to $SU(N)$

Consider the $SU(N)$ spin chain with p-box rep. on each site.

Nonlinear sigma model

Under certain assumptions, Bykov ('12,'13) and Lajko et al. ('17) showed that the LEFT is $SU(N)/U(1)^{N-1}$ NLσM:

$$S = \frac{1}{g^2}\sum_{i=1}^{N}\int |(\mathrm{d}+\mathrm{i}a_i)z_i|^2 + \mathrm{i}\sum_{i=1}^{N}\frac{\theta_i}{2\pi}\int \mathrm{d}a_i + \cdots .$$

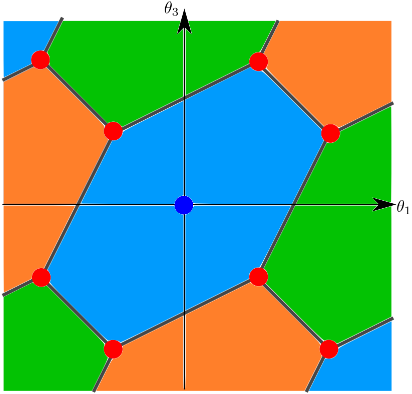
Here, $U=[z_1,\ldots,z_N]$ is an $SU(N)$ matrix, and

$$\theta_k = \frac{2\pi p}{N}k.$$

Using this result, we constrain the phase diagram by anomaly and global inconsistency matching, and generalize the Haldane conjecture.

Phase diagram of $SU(3)/U(1)^2$ NLσM

Before showing the computation, let us show the result:



- Different colors = different SPT phases (by global inconsistency)
- Red blobs = $SU(3)_1$ WZW model, or trimerized phase (by anomaly matching)

Symmetries of $SU(3)/U(1)^2$ NLσM

$PSU(3)$ spin symmetry

$z_i \mapsto V z_i$ with $V \in SU(3)$, coming from the spin rotation $\boldsymbol{S}_i \mapsto V \boldsymbol{S}_i V^{-1}$.

$\mathbb{Z}_3$ permutation symmetry

$z_i \mapsto z_{i+1}$ and $a_i \mapsto a_{i+1}$, coming from lattice translation $\boldsymbol{S}_i \mapsto \boldsymbol{S}_{i+1}$. This symmetry exists only for special theta angles, such as $\theta_i = \frac{2\pi p}{3} i$. (Red and blue blobs in the Figure)

Parity, P_k

$\theta_i \mapsto -\theta_{k-i}$, coming from $\boldsymbol{S}_i \mapsto \boldsymbol{S}_{k-i}$.
This exists only for special theta's, satisfying $\theta_i + \theta_{k-i} = 0 \bmod 2\pi$. (Gray lines in the Figure)

Gauging $PSU(3)$ symmetry

We will see that the spin rotation and lattice translation cannot be gauged simultaneously (= mixed 't Hooft anomaly).

To gauge $PSU(3)$, we introduce (cf. Kapustin, Seiberg, 2014)

- A: $U(3)$ gauge field,
- B: $U(1)$ two-form gauge field, with $3B = \mathrm{d}(\mathrm{tr}(A))$.

The gauged action is given by

$$S[(A,B)] = \frac{1}{g^2}\sum_{i=1}^{3}\int |(\mathrm{d}+\mathrm{i}a_i+\mathrm{i}A)z_i|^2 + \mathrm{i}\sum_{i=1}^{3}\frac{\theta_i}{2\pi}\int(\mathrm{d}a_i+B)+\cdots .$$

$\Rightarrow$ 2π periodicity of theta's becomes $2\pi N (= 6\pi)$.

$PSU(3)$-$\mathbb{Z}_3$ 't Hooft anomaly

Define the partition function $Z[A,B]$ with the $PSU(3)$ gauge field.

$\mathbb{Z}_3$ lattice translation gives

$$Z[A,B] \mapsto Z[A,B] \exp\left(\mathrm{i}p\int B\right).$$

This phase is nontrivial iff $p \neq 0 \bmod 3$, and gives 't Hooft anomaly.
't Hooft anomaly matching is satisfied by

- SSB of $\mathbb{Z}_3$ lattice translation (trimerized phase), or
- Conformal field theory.

Second choice gives the $SU(3)$ version of the Haldane conjecture.

(Affleck, Lieb, '86)

$SU(3)$ Wess-Zumino-Witten model

The level-k $SU(3)$ WZW model is defined by

$$S = \frac{k}{8\pi^2}\int_{M_2} |\mathrm{d}\mathcal{U}|^2 + \frac{\mathrm{i}k}{12\pi}\int_{M_3} \mathrm{tr}[(\mathcal{U}^{-1}\mathrm{d}\mathcal{U})^3].$$

The model has the chiral symmetry, and we pay attention to its subgroup:

$$\frac{SU(3)_L \times SU(3)_R}{\mathbb{Z}_3} \supset PSU(3)_V \times (\mathbb{Z}_3)_L.$$

We can show that it has the same anomaly of $SU(3)/U(1)^2$ model if

$$\mathrm{gcd}(3,p) = \mathrm{gcd}(3,k).$$

$\Rightarrow$ Combined with the C-theorem, we conjecture that the anomaly is matched by $SU(3)_1$ WZW.

$PSU(3)$-P global inconsistency

We take two parity symmetry point, such as $(\theta_1, \theta_3) = (0, 0)$ and $(2\pi, 0)$. Then,

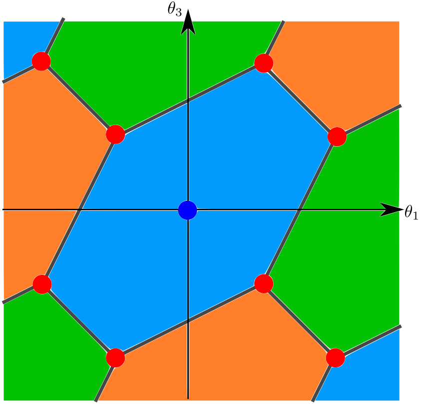
$$\begin{aligned} \mathrm{P} \quad : \quad & Z_{(0,0)}[A, B] \mapsto Z_{(0,0)}[A, B], \\ & Z_{(2\pi,0)}[A, B] \mapsto Z_{(2\pi,0)}[A, B] \exp\left(-2\mathrm{i} \int B\right). \end{aligned}$$

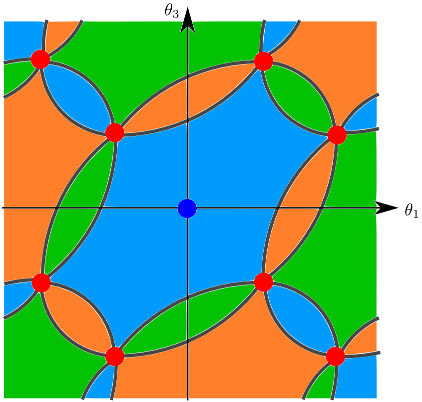
The second one is not anomaly, because we can cancel it by a counter term $\mathrm{i} \int B$.

This is the global inconsistency, i.e. we cannot gauge P for both points with the same counterterm (Gaiotto, Kapustin, Komargodski, Seiberg, '18). This is matched by (Tanizaki, Kikuchi, '18)

- Those two points are different symmetry-protected topological orders protected by $PSU(3)$.
- One of them has nontrivial ground states.

Two possible phase diagrams of $SU(3)/U(1)^2$ NLσM





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Summary

- Anomaly matching provides a strong constraint on nonperturbative physics.
- We almost completely determined the phase structure of $SU(N)$ spin chains just by symmetry, anomaly, and global inconsistency.
- Haldane conjecture is generalized to $SU(N)$ spin chains: $SU(N)$ antiferromagnetic spin chain with p-box rep. is described by the level-$\mathrm{gcd}(N,p)$ $SU(N)$ Wess-Zumino-Witten model.